Attractive Casimir effect in an infrared modified gluon bag model

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In this work, we are motivated by previous attempts to derive the vacuum contribution to the bag energy in terms of familiar Casimir energy calculations for spherical geometries. A simple infrared modified model is introduced which allows studying the effects of the analytic structure as well as the geometry in a clear manner. In this context, we show that if a class of infrared vanishing effective gluon propagators is considered, then the renormalized vacuum energy for a spherical bag is attractive, as required by the bag model to adjust hadron spectroscopy.

I. INTRODUCTION

In the MIT bag model, a hadron is regarded as a finite region of space where quark and gluon fields are confined. The bag energy necessary to fit hadron masses must contain a term of the form $-Z/R$, where R is the bag radius and Z is a phenomenological positive constant of order one [1]. It is commonly accepted that this term is essentially a manifestation of zero point energies of the confined fields, however an understanding of this effect in the context of Casimir energy calculations remains intriguing.

These calculations consider gluons as free massless particles inside the bag, an assumption based on asymptotic freedom. Boyer, Davies, Bender and Hays [2, 3, 4], and many others, studied massless scalar, vector and fermionic fields assuming they are confined in a spherical region of space. A systematic study has been developed by Milton [5, 6, 7]. In all these studies, the renormalized zero-point energy contribution to the MIT bag energy contains the term $\sim 1/R$, but with a negative Z (≈ -0.7), that is, a repulsive effect is obtained, instead of the attractive one necessary to adjust hadron spectroscopy. To the best of our knowledge, there is no explanation in the literature for this discrepancy. For a discussion of the bag model and the current status of Casimir calculations in the bag see refs. [8] and [9], respectively.

A resolution to the sign problem is hard to accomplish as in general there is no intuition whether the Casimir energy should be positive or negative in a given situation. In general, the sign of the Casimir energy may depend on spacetime dimensionality, the type of boundary conditions, the shape of the boundaries, the spin of the field, etc. For a discussion of these issues, see for example refs. [10] [11] [12].

On the other hand, any treatment of QCD with boundaries requires simplifying assumptions to render the calculation possible. In this regard, we note that the above mentioned simple model considering free massless gluons inside the bag only encodes nonperturbative information by imposing boundary conditions on the bag boundary, while the free propagator is taken over the whole range of momenta that can be accommodated inside the bag. This means, momenta ranging typically from the inverse bag radius up to the ultraviolet region. We would like to stress that infrared modes are more sensitive to the bag geometry, and these are precisely modes in the nonperturbative region of QCD. This situation, shows that any improvement to obtain an attractive Casimir energy must take into account intrinsic nonperturbative effects inside the bag such as the analytic structure QCD.

At present, a derivation of the Casimir energy from first principles seems hopelessly out of reach. In this work we would like to propose instead a possible understanding of the attractive nature of vacuum fluctuations, based on a simple bag model extended to incorporate an infrared modified gluon propagator. The associated infrared behavior will be motivated by recent studies on the analytic structure of the confined gluon propagator in pure QCD [13].

II. SIMPLE INFRARED MODIFIED BAG MODEL

In QCD it is believed that colored objects such as quarks and gluons cannot exist in asymptotic states. They should give place instead to the hadronic spectrum. These open problems are associated with a nonperturbative regime, and supposed to be driven by the dynamical generation of a mass scale, which separates the pertur-

bative and nonperturbative phases.

In the bag model, the boundary condition is devised to represent the nonperturbative physics associated with the finite size of colorless objects such as hadrons.

Now, regarding the renormalized vacuum energy, if it is computed by considering the gluon sector only (quenched model), the inclusion of additional nonperturbative information is a sensible modification to be considered.

In this regard, note that such a calculation would be based on pure QCD proved by means of a boundary condition on a sphere of radius R. Of course, for very small R, the effect of the boundaries would be relevant, while, because of asymptotic freedom, the physics inside the bag would essentially be perturbative. On the other hand, for very large R, the model would behave as pure QCD without boundaries, displaying the intrinsic nonperturbative behavior of the gluon sector.

For typical values of R associated with nonperturbative objects such as hadrons, both effects should be relevant when computing the renormalized vacuum energy: finite size effects representing the hadron boundaries, as well as intrinsic nonperturbative behavior in the gluon sector. In particular, possible nonperturbative analytic properties in this sector constitute an interesting aspect to be explored.

In this section, according to the discussion above, an infrared modified bag model is presented. The basic physical inputs are the following:

• As usual, we will consider the hadron as a finite region in space where the fields are confined by imposing a boundary condition at the bag boundary.

• Additional intrinsic nonperturbative information will be parametrized in a model gluon propagator $G(k^2)$ associated with an effective quadratic gluon action inside the bag. Although this procedure cannot be justified from first principles, similar ideas have been already used in different hadron models where some interesting results have been obtained [14]. This enables the computation of the vacuum energy by means of the usual formula,

$$
\begin{aligned}
E_C &= \frac{i}{2T} \mathrm{tr}\ln G \\
&= \frac{i}{2}\sum_n \int \frac{dk_0}{2\pi} \ln G(k_0, k_n),
\end{aligned}
\tag{1}
$$

with $G(k^2)$ in the place of the free gluon propagator $1/(k^2 + i\epsilon)$. In this equation, T is the infinite time that the configuration exists and k_n^2 refers to the eigenvalues of the Laplace operator in the corresponding geometrical configuration. As in former studies, we are considering a scalar Casimir energy to be multiplied by the number of polarization modes of the gluon field (including color) (see ref. [9]).

• The model gluon propagator $G(k^2)$ incorporates nonperturbative effects such as pole suppression. That is, we

will change the free $1/(k^2 + i\epsilon)$ ($k^2 = k_0^2 - \mathbf{k}^2$) propagator used in previous works, which is valid in the ultraviolet region due to asymptotic freedom, by an infrared modified one. After a Wick rotation $k^2 + i\epsilon \to -\bar{k}^2$, the behavior of the modified propagator can be defined in euclidean $\bar{k}$ momentum space,

$$
G(-\bar{k}^2) \sim \begin{cases} \mathcal{R}(\bar{k}^2)\,(1/\bar{k}^2), & \bar{k}^2 \text{ large} \\ \mathcal{R}(0)\,(\bar{k}^2)^\lambda, & \lambda > 0, \ \bar{k}^2 \text{ small.} \end{cases}
\tag{2}
$$

This type of infrared vanishing behavior has been obtained in different scenarios (see the discussion below). For large $\bar{k}^2$, we have also included a possible $\mathcal{R}(\bar{k}^2)$ factor encoding running coupling constant information. Note also that the free propagator would correspond to $\lambda = -1$ and $\mathcal{R} \equiv 1$.

When computing Casimir energies, we believe that the above mentioned simple model can capture the essential modifications implied by the analytic structure of QCD. With regard to the behavior in (2), we are motivated by recent progress on the form of the gluon propagator in pure QCD. In general, because of confinement, we know that the gluon propagator must suffer a dramatic change when $k^2 \to 0$: as gluons cannot appear in asymptotic states, the free pole at $k^2 = 0$ must disappear in the complete theory. In fact, many authors coincide that the exact propagator should be infrared finite when $k^2 \to 0$. Indeed, an infrared vanishing behavior has been obtained in different scenarios: by studying the Schwinger-Dyson equation [15], by restricting the path integral so as to avoid Gribov copies [16], and also in a Lattice formulation [17]. In other words, different scenarios point to an intrinsic analytic structure in pure QCD associated with an infrared suppressed propagator, describing the so called "confined" gluons, in contrast to an infrared enhanced "confining" $1/k^4$ behavior (for a review, see ref. [13]).

In particular, studies of the (Landau gauge) Schwinger-Dyson equation in pure QCD lead to an infrared vanishing gluon propagator $G(k^2)(\eta^{\mu\nu} - k^\mu k^\nu/k^2)$, with the form [15],

$$
G(k^2) = (-k^2 - i\epsilon)^\lambda (-k^2 + \Lambda^2 - i\epsilon)^{-(\lambda+1)} \mathcal{R}(-k^2 - i\epsilon),
\tag{3}
$$

where $\lambda > 0$ and Λ is of the order of Λ_{QCD}. Note that after a Wick rotation, $-k^2 - i\epsilon \to \bar{k}^2$, this ansatz satisfies the infrared vanishing behavior given in (2). In the Schwinger-Dyson context, the additional condition $\lambda < 0.4$ is satisfied, and the factor $\mathcal{R}(\bar{k}^2)$ represents the perturbative running. That is, for $\bar{k}^2 >> \Lambda^2$,

$$
\mathcal{R}(\bar{k}^2) = \alpha^{-\gamma} \quad , \quad \alpha \sim \frac{\mathrm{const.}}{\ln(\bar{k}^2/\Lambda^2)} \quad , \quad -\gamma > 0.
\tag{4}
$$

On the other hand, the Gribov mechanism is based on the restriction of the path integral domain so as to avoid copies of the gauge fields in the pure gluon theory defined on euclidean space. This is necessary as in the nonperturbative region the usual gauge conditions do not fix

the gauge completely. In Landau gauge, the associated nonperturbative gluon propagator is given by,

$$G_{\text{Gribov}}(-\bar{k}^2) = \frac{\bar{k}^2}{\bar{k}^4 + M^4}, \qquad (5)$$

which satisfies the behavior (2) with $\lambda = 1$ and $\mathcal{R} \equiv 1$.

As we are concerned with a model gluon propagator, we will first develop a Casimir energy calculation based on the associated general analytic properties displayed in Minkowski space, not relying on a specific form for the propagator. We will obtain a representation for the Casimir energy where the analytic structure of the model and the effects of geometry will be clearly distinguished. Only at the end we will discuss the effect of pole suppression on the renormalized vacuum energy. For this aim, we will consider for instance a model gluon propagator of the form given in eq. (3), which displays a well defined analytic structure in Minkowski space. In this regard, note that the Gribov ansatz cannot be rotated back to Minkowski space because of the complex poles in eq. (5); as is well known, the Gribov scenario is only defined in euclidean space.

III. CASIMIR ENERGY AND THE ANALYTIC STRUCTURE OF G

In order to study the effect implied by the analytic structure of the modified propagator, we will start by representing the logarithmic derivative of G in terms of the decomposition,

$$\frac{d}{d\mathcal{A}} \ln G(\mathcal{A} + i\epsilon) = \int_0^\infty d\mu^2 \beta(\mu^2) \frac{1}{-\mathcal{A} + \mu^2 - i\epsilon}, \quad (6)$$

where $\mathcal{A} = k^2$ and β is given by,

$$\beta(k^2) = \frac{1}{2\pi i} \left(\frac{d}{d\mathcal{A}} \ln G(\mathcal{A} + i\epsilon) - \frac{d}{d\mathcal{A}} \ln G(\mathcal{A} - i\epsilon) \right). \tag{7}$$

Defining the jump at the discontinuity of an analytic function $F(\mathcal{A})$,

$$\delta[F(\mathcal{A})] \equiv F(\mathcal{A} + i\epsilon) - F(\mathcal{A} - i\epsilon), \qquad (8)$$

we can also write,

$$\beta(k^2) = \frac{1}{2\pi i} \delta \left[\frac{d}{d\mathcal{A}} \ln G(\mathcal{A}) \right]. \qquad (9)$$

In this manner, from eq. (6), we have the representation,

$$\ln G(\mathcal{A} + i\epsilon) = C + \int_0^\infty d\mu^2 \beta(\mu^2) \ln G_{\mu^2}(\mathcal{A} + i\epsilon), \quad (10)$$

with $C = \text{const.}$ and $G_{\mu^2}(\mathcal{A}) = (-\mathcal{A} + \mu^2)^{-1}$ being the free propagator for a field with mass parameter μ^2.

In fact, when defining the Casimir energy in eq. (1), a parameter χ^2 with the dimension of [mass]^2 multypling

G must be introduced, so as to have a dimensionless argument in the logarithm. In an equivalent manner, we can introduce a parameter ξ^2, $[\xi] = \text{mass}$, multiplying G_{μ^2} in the second member of eq. (10), and absorb the constant C in its definition; then, we have,

$$\ln \left(\chi^2 G \right) = \int_0^\infty d\mu^2 \beta(\mu^2) \ln \left(\xi^2 G_{\mu^2} \right). \qquad (11)$$

Now, considering this replacement in eq. (1), we can take the formal trace in the second member of eq. (11), to obtain the Casimir energy in terms of the regularized representation,

$$E_C = \int d\mu^2 \beta(\mu^2) \frac{i}{2} \sum_n \int \frac{dk_0}{2\pi} \ln \left(\xi^2 G_{\mu^2} \right) \bigg|_{\text{reg}}. \tag{12}$$

In other words we can write,

$$E_C = \int d\mu^2 \beta(\mu^2) E_C(\mu^2), \qquad (13)$$

where

$$E_C(\mu^2) = \frac{i}{2} \sum_n \int \frac{dk_0}{2\pi} \ln \left(\xi^2 G_{\mu^2}(\mathcal{A}_n + i\epsilon) \right) \bigg|_{\text{reg}} \qquad (14)$$

is the Casimir energy for free field modes with mass parameter μ^2, obtained by means of the regularized partition function $(\mathcal{A}_n = k_0^2 - k_n^2)$. After a Wick rotation $k_0 \to i\bar{k}_0$, we can also write,

$$E_C(\mu^2) = \frac{1}{2} \sum_n \int \frac{d\bar{k}_0}{2\pi} \ln \left(\frac{\bar{k}_0^2 + k_n^2 + \mu^2}{\xi^2} \right) \bigg|_{\text{reg}}, \quad (15)$$

which can be defined, for instance, by using the zeta function regularization technique,

$$E_C(\mu^2) = -\frac{1}{2} \frac{d}{ds} \zeta_4(s) \bigg|_{s=0}, \qquad (16)$$

$$\zeta_4(s) = \sum_n \int \frac{d\bar{k}_0}{2\pi} \left(\frac{\bar{k}_0^2 + k_n^2 + \mu^2}{\xi^2} \right)^{-s}. \qquad (17)$$

As discussed in ref. [18], the zeta function result for $E_C(\mu^2)$ in eq. (16) differs in general from the Casimir energy obtained by the zeta function regularization of the summation of the free field energy modes, $E_{\text{mode}} = (1/2) \sum_n (k_n^2 + \mu^2)^{1/2}|_{reg}$,

$$E_{\text{mode}}(\mu^2) = \frac{\xi}{2} \lim_{\epsilon \to 0} \frac{1}{2} \left[\zeta_3(-1/2 + \epsilon) + \zeta_3(-1/2 - \epsilon) \right], \tag{18}$$

$$\zeta_3(s) = \sum_n \left(\frac{k_n^2 + \mu^2}{\xi^2} \right)^{-s}. \qquad (19)$$

In that reference, the following relationship has been established,

$$E_C(\mu^2) = E_{\text{mode}}(\mu^2) + \frac{1}{2}\left(\psi(1) - \psi(-1/2)\right)\frac{\xi C_2(\xi)}{4\pi^2},$$
(20)

where $\psi(s)$ is the digamma function and C_2 is the second Seeley-De Witt coefficient.

While the C_2 coefficient for a geometry consisting of two parallel plates vanishes, in the case of a spherical bag it is nonzero. As noted in ref. [18], this difference reflects the inherent ambiguity introduced in the Casimir energy when removing the polar part in eq. (19), by choosing the principal value, such as in eq. (18), or by using other possible prescriptions.

According to ref. [18], C_2 can be read from the variation of E_{mode} under a change of the parameter ξ,

$$E_{\text{mode}}|_{\xi'} - E_{\text{mode}}|_\xi = -\frac{\xi C_2(\xi)}{4\pi^2}\ln\frac{\xi'}{\xi}.$$
(21)

We will discuss below the particular form of C_2 in the bag model context, and we will see that both Casimir energy definitions, E_C and E_{mode}, lead in fact to the same physical implications.

It is important to remark that in the superposition we have derived for the Casimir energy E_C (cf. eq. (13)), the Casimir energies $E_C(\mu^2)$ encode the geometry, while the factor $\beta(\mu^2)$ encodes the analytic structure of the model. Note also that the propagator for a free massive field of mass m would give $\beta(\mu^2) = \delta(\mu^2 - m^2)$ in eq. (6), and eq. (13) would give $E_C = E_C(m^2)$, as expected.

In general, because of the presence of $E_C(\mu^2)$, our expression corresponds to a physical situation where all the μ^2 field modes that participate in the representation (6) are confined, that is, they see the boundary condition. From this point of view our approach is quite natural, since confinement is supposed to act over all the gluon modes represented by the model propagator G. This should be contrasted with typical radiative QED corrections to the Casimir effect where, in the computation strategy, the boundary condition is seen by the free photon propagator but not by the continuum of electron-positron modes represented in the vacuum polarization [19]. In these cases the overall result is a renormalization of the sphere radius, the distance between plates, etc. This renormalization can be traced back to the effect of electron-positron pairs, which are not confined by the boundary conditions, thus leading to an effective enlargement of the confining region.

We also note that since our formula relies on the superposition of the usual Casimir energies, obtained from the regularized partition function for field modes with mass parameter μ^2, the form of the divergences we will obtain here are the standard ones. For a detailed discussion see refs. [20, 21, 22]. These divergent terms will renormalize similar terms in the bag model to be fixed by phenomenology. For instance, the divergent volume term will renormalize the bag constant B.

Now, according to our discussion at the end of section §II, let us consider for the model gluon propagator an ansatz of the form given in eq. (3). In this case, eq. (9) leads to,

$$\begin{aligned}\beta(k^2) &= \frac{1}{2\pi i}\delta\left[\frac{\lambda}{k^2} - \frac{1+\lambda}{k^2 - \Lambda^2} + \frac{d\ln\mathcal{R}(\mathcal{A})}{d\mathcal{A}}\right]\\ &= -\lambda\delta(k^2) + (1+\lambda)\delta(k^2 - \Lambda^2) + \beta_\mathcal{R}(k^2),\end{aligned}$$
(22)

where we have used eq. (8), the property

$$1/(k^2 + i\epsilon) - 1/(k^2 - i\epsilon) = -2\pi i\,\delta(k^2),$$

and the definition,

$$\beta_\mathcal{R}(k^2) = \frac{1}{2\pi i}\delta\left[\frac{d\ln\mathcal{R}(\mathcal{A})}{d\mathcal{A}}\right].$$
(23)

Then, from eqs. (13) and (22), the Casimir energy for the infrared modified model results,

$$E_C = -\lambda E_C(0) + (1+\lambda)E_C(\Lambda^2) + \int_0^\infty d\mu^2\,\beta_\mathcal{R}(\mu^2)E_C(\mu^2).$$
(24)

Of course, if a free massless propagator were considered, that is, $\lambda = -1$ and $\mathcal{R} \equiv 1$ (cf. eq. (3)), we would have $\beta(\mu^2) = \delta(\mu^2)$, and the Casimir energy $E_C = E_C(0)$ for a massless field would be reobtained.

In that case, the zeta function calculation for the regularized energy mode summation in eq. (18) has been performed in ref. [20], obtaining,

$$E_{\text{mode}}(0) = \frac{1}{R}\left[0.08392 + \frac{8}{315\pi}\ln(\xi R)\right],$$
(25)

which corresponds to photon modes confined inside the bag. Using this expression in eq. (21), we read,

$$\frac{\xi C_2(\xi)}{4\pi^2} = -\frac{8}{315\pi}\frac{1}{R},$$
(26)

which corresponds to an additional $1/R$ term when passing from the Casimir energy defined by $E_{\text{mode}}(0)$ to $E_C(0)$ in eq. (20); evaluating the digamma functions, we get,

$$E_C(0) = \frac{1}{R}\left[0.08640 + \frac{8}{315\pi}\ln(\xi R)\right].$$
(27)

In ref. [20], the zeta function result for the summation of energy modes in eq. (25) has been compared with that obtained in ref. [7],

$$\frac{1}{R}\left[0.08984 + \frac{8}{315\pi}\ln(\delta/8)\right],$$
(28)

found by the Green's function method, where δ is a cutoff associated with a nonzero "skin depth" representing a realistic boundary, instead of a sharp mathematical one.

As noted in that reference, the $1/R$ parts obtained from the first term in eqs. (25) and (28), differ within a 6.7%, and in fact, there is no reason why these parts should be equal, as they may vary by just changing the values of the parameters δ and ξ. The same situation applies to the $1/R$ part coming from E_C, obtained from the first term in eq. (27).

In a bag model context, the point is that a natural scale exists which permits to derive some qualitative and semiquantitative conclusions (see ref. [9] and references therein).

For instance, the plausible value for $\ln(\delta/8)$ is of order one, when realistic boundary conditions with a "skin depth" of about 10% of a typical bag radius is considered, see ref. [9]. There, a first crude estimate $\sim +0.7/R$ has been obtained for the zero point energy by dropping the logarithm term in eq. (28) (which corresponds to 10% of the first term), and multiplying by a factor eight counting for the number of gluons. As stated in [9], it is certainly very hard to doubt about the sign of the effect.

In the zeta function regularization scenario, according to refs. [20], [9] and [21], in a pure QCD context ξ should be associated with the energy scale parameter Λ_{QCD}; as a consequence, we will see that the discussion in the above paragraph is also realized in this type of scenario.

Let us consider $\xi = \Lambda = 600\,\mathrm{Mev}$ (a typical value for the fitting of the nonperturbative propagator (3)), and the radius R taking values around $R_0 = \frac{1}{200}\,\mathrm{Mev}^{-1}$ (a typical value for a hadron radius). We see that for this choice the logarithm in eqs. (25) and (27) is of order one, and again the corresponding second term represents about 10% of the first term. In general, we can write eq. (27) as,

$$E_C(0) = \frac{1}{R}\left[0.08640 + \frac{8}{315\pi}\ln(\Lambda R_0) + \frac{8}{315\pi}\ln(R/R_0)\right].\tag{29}$$

After multiplying by eight colors, the first two terms in the bracket give a contribution $+0.75/R$. Then, we see that in the zeta function regularization scenario, a clear scale for the Casimir effect again arises. The Casimir energy in the "photon-like" bag context can be considered as consisting of a $\sim +0.7/R$ part, plus logarithmic corrections.

We also note that even in an R interval that covers two orders of magnitude around R_0, from $(1/10)\,R_0$ to $10\,R_0$, the whole bracket in eq. (29) remains of order one, varying from $+0.6$ to $+0.9$. Then, in this scenario it is also very hard to doubt about the repulsive character of the effect. For "photon-like" gluon modes confined inside the bag, in order to change the sign of the Casimir energy and comply with phenomenology, the typical hadron radius R_0 and typical scale Λ in pure QCD should lead to a negative contribution of order one coming from the first two terms in eq. (29). This would happen for $\Lambda R_0 < 10^{-9}$, which is an unrealistic situation.

Coming back to the infrared modified bag model, we observe that the first term in eq. (24) only involves the soft $\mu^2 = 0$ modes. On the other hand, note that the second term in eq. (24) is associated with a massive Casimir effect $E_C(\Lambda^2)$ which is expected to be suppressed.

This can be seen by considering the general form of the Casimir effect for a massive scalar field. In ref. [22], the zeta function regularized expression for the Casimir energy has been computed; the associated divergent terms and ambiguities always display nonnegative powers of the mass of the field. On the other hand, as discussed in ref. [21], in the massive case there is a natural renormalization prescription, requiring that in the infinite mass limit all effects coming from quantum fluctuations should vanish. In this manner, after renormalization, it is obtained,

$$E_C(\Lambda^2) = \frac{f(\Lambda R)}{R},\tag{30}$$

where f is a well defined, finite and unambiguous function.

Although a simple analytic expression for $f(\Lambda R)$ is lacking, its numerical evaluation has been performed in ref. [22], and the result for the interior problem with Dirichlet boundary conditions has been presented by plotting $f = RE_C$ as a function of the dimensionless variable ΛR. By using this information, it is simple to estimate the second term in eq. (24), taking as before $\Lambda = 600\,\mathrm{Mev}$, and $R = \frac{2}{3}\frac{1}{200}$ or $R = \frac{1}{200}$ (typical values in Mev^{-1} for the pion and proton radius, respectively). Around these values, we have $E_C(\Lambda^2) = f(2)/R$ or $f(3)/R$, respectively. From ref. [22], we can extract the estimate $f(2) \approx .0002$ which, after including a 2×8 factor counting the number of physical polarizations times the number of colors, implies that the second term in eq. (24) typically corresponds to a 1% of the first term (λ is of order 1). The analysis for $f(3)$ gives an even higher suppression than the previous one.

Note that in our model the first two factors in eq. (3) are the relevant ones to interpolate between an ultraviolet asymptotically free and an infrared vanishing behavior of the effective gluon propagator. So that, in principle, the model could be defined with $\mathcal{R} \equiv 1$ and the third term in eq. (24) would be absent.

However, it is interesting to analyze the effect of a nontrivial $\mathcal{R}$ factor. To be more specific, let us consider for instance the ansatz for $\mathcal{R} = \alpha^{-\gamma}$ (cf. eq. 4), now extended over the whole range of momenta according to [15],

$$\mathcal{R} = (\mathcal{A} - \Lambda^2)^\gamma \alpha'^{-\gamma},\tag{31}$$

$$\alpha' = -\alpha(0)\Lambda^2 + \frac{4\pi}{b}\mathcal{A}\left(\frac{1}{\ln(-\mathcal{A}/\Lambda^2)} + \frac{\Lambda^2}{\mathcal{A} + \Lambda^2}\right),\tag{32}$$

where $\alpha(0) = 8.915/N_c$, $\gamma = (-13N_c + 4N_f)/(22N_c - 4N_f)$, $b = (11N_c - 2N_f)/3$, N_c and N_f being the number of colors and flavors, respectively, and the $i\epsilon$ prescription in the variable $\mathcal{A} = k^2$ is understood. This form implies a cut in Minkowski space along the timelike k^2

axis with branch point at $k^2 = 0$, and the contribution to the discontinuity $\beta_{\mathcal{R}}(\mu^2)$ defined in eq. (23) has support starting at $\mu^2 = 0$.

Using eq. (23), we have,

$$\beta_{\mathcal{R}}(k^2) = -\gamma\delta(k^2 - \Lambda^2) + \beta'_{\mathcal{R}}(k^2),$$

$$\beta'_{\mathcal{R}}(k^2) = -\frac{\gamma}{2\pi i}\,\delta\left[\frac{d}{d\mathcal{A}}\ln\alpha'\right]. \qquad (33)$$

The first term in $\beta_{\mathcal{R}}$ simply renormalizes the second term in eq. (24), that is,

$$E_C = -\lambda E_C(0)+$$
$$+(1 + \lambda - \gamma)E_C(\Lambda^2) + \int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2)E_C(\mu^2), \qquad (34)$$

and as γ is of order one, the same analysis we have done before can be applied to conclude that the second term in eq. (34) is suppressed.

On the other hand, $\beta'_{\mathcal{R}}$ can be written in terms of the real and imaginary parts of $\alpha'(\mathcal{A} + i\epsilon)$,

$$\beta'_{\mathcal{R}}(k^2) = -\frac{\gamma}{\pi}\frac{d}{d\mathcal{A}}\arctan\left(\frac{\Re(\alpha')}{\Im(\alpha')}\right), \qquad (35)$$

$$\Re(\alpha') = -\alpha(0)\Lambda^2 + \frac{4\pi}{b}\mathcal{A}\left(\frac{\ln(\mathcal{A}/\Lambda^2)}{\ln^2(\mathcal{A}/\Lambda^2) + \pi^2} + \frac{\Lambda^2}{\mathcal{A} + \Lambda^2}\right), \qquad (36)$$

$$\Im(\alpha') = \frac{4\pi^2}{b}\frac{\mathcal{A}}{\ln^2(\mathcal{A}/\Lambda^2) + \pi^2}. \qquad (37)$$

In Fig. 1, we plot $\beta'_{\mathcal{R}}(\mu^2)$ as a function of μ. We observe that the function $\beta'_{\mathcal{R}}$ displays a definite sign, weighting the massive Casimir energies in the third term of eq. (34).

The total weight that is distributed over the whole range of masses is,

$$\int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2) = -\frac{\gamma}{\pi}\,\arctan\left(\frac{\Re(\alpha')}{\Im(\alpha')}\right)\Big|_0^\infty = -\gamma, \quad (38)$$

which can also be verified numerically; for $N_c = 3$, and $N_f = 0$ (quenched theory), we have $-\gamma \approx 0.59$.

Now, we can use the general parametrization of the Casimir energy in the massive case, given in eq. (30), with $\Lambda \to \mu$, to write,

$$\int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2)E_C(\mu^2) = \frac{1}{R}\int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2)f(\mu R). \qquad (39)$$

In order to obtain a bound for the contribution of this term, we can consider, as before, a typical radius $R = \frac{1}{200}\,\mathrm{Mev}^{-1}$, and divide the μ^2 integration in three intervals I_i, $i = 0, 1, 2$: from 0 to $(200\,\mathrm{Mev})^2$, $(200\,\mathrm{Mev})^2$ to $(400\,\mathrm{Mev})^2$ and $(400\,\mathrm{Mev})^2$ to ∞.

Note also in Fig. 1 that in the limit $\mu^2 \to 0$, we have $\beta'_{\mathcal{R}} \to 0$. Indeed, it can be verified that $\beta'_{\mathcal{R}}$ tends to zero
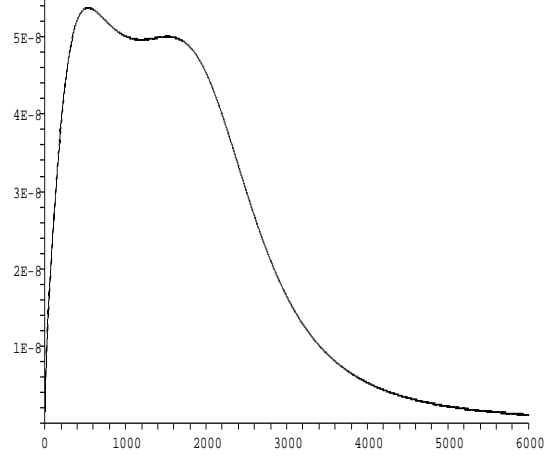

FIG. 1: This is a plot of $\beta'_{\mathcal{R}}$ (in Mev^{-1}), given in eq. (35), vs. the mass parameter μ (in Mev). We have considered $N_c = 3$, and $N_f = 0$ (quenched case).

like $\sim 1/(\ln\mu^2)^2$. Therefore, in eq. (39), the contribution coming from a region very close to $\mu^2 = 0$ is suppressed. Outside from this small region, it has been shown in ref. [22] that $f(\mu R)$ is positive definite, displaying a maximum value $f_0 \approx .0030$ in the region I_0. In the region $\mu R \geq 1$, the function $f(\mu R)$ decreases monotonically to zero. Then, the maximum values in regions I_1 and I_2 are given by $f_1 = f(1) \approx .0005$ and $f_2 = f(2) \approx .0002$, respectively. These values have been estimated from the numerical result presented in that reference.

Then, replacing in each interval $f(\mu R)$ by f_i, we obtain,

$$\int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2)f(\mu R) \leq \sum_{i=0}^2 f_i\int_{I_i} d\mu^2\,\beta'_{\mathcal{R}}(\mu^2) \qquad (40)$$

With regard to the integrals in eq. (40), we obtained .001 and .006 in the intervals I_0 and I_1, while in interval I_2 the integral completes the total weight .59. In this manner, around a typical radius of $R = \frac{1}{200}\,\mathrm{Mev}^{-1}$,

$$\int_0^\infty d\mu^2\,\beta'_{\mathcal{R}}(\mu^2)E_C(\mu^2) \leq 10^{-4}/R. \qquad (41)$$

After including a factor for the total number of polarizations, we see that this term represents a contribution less than 0.3% when compared with the first term in eq. (34).

As expected, the effect of the boundaries on the finite part of the zero point energy is dominated by the soft modes $\mu^2 = 0$ present in the first term of eq. (34). We have also considered a number of flavors $N_f = 3$, obtaining no essential modifications in the analysis.

Then, in the context of the infrared modified bag model, the $1/R$ part of the renormalized vacuum energy turns out to be $-Z/R$, with $Z \sim +0.7\lambda$, which is attractive for an infrared vanishing model propagator ($\lambda > 0$).

IV. CONCLUSIONS

In this article, we have computed the Casimir energy in a bag model containing a modified gluon propagator. Our motivation comes from previous attempts to describe the vacuum energy in the bag in terms of Casimir energy calculations. These attempts, based in a model where the gluons are associated with massless fields confined on a spherical region, failed to describe the attractive nature of the vacuum energy necessary to adjust hadron spectroscopy.

In the modified framework we have considered, we were able to clearly trace the effects introduced by the analytic structure of the model, on the one hand, and the geometry of the boundaries, on the other (see equation (13)). In this context, we can see that the introduction of perturbative information in the above mentioned massless gluon models is not enough to render the finite part of the zero point energy attractive; if only a running coupling were taken into account, the effect would be suppressed when compared with the main Casimir effect coming from the pole at $k^2 = 0$, which is repulsive.

Then, in order to understand the attractive nature of the Casimir energy due to gluons, the consideration of intrinsic nonperturbative effects is fundamental, however, an analytic approach from first principles is a formidable challenge. For this reason, we have introduced the simple infrared modified bag model with an effective gluon propagator. Within this context, we have seen that the repulsive or attractive nature of the vacuum energy depends on whether the model propagator describes "confined" gluons with infrared vanishing behavior, like in the Schwinger-Dyson and Gribov scenarios, or it is singular at $k^2 = 0$. This last situation is verified, for instance, when a free $1/k^2$ or a "confining" $1/k^4$ model gluon propagator is used.

From this point of view, we see that an infrared vanishing gluon propagator is preferred as it corresponds to an attractive term $\sim -0.7\lambda/R$, $\lambda > 0$, in the renormalized vacuum energy. Hadron phenomenology requires a gluon contribution of the form $-Z/R$, with Z of order one (see the discussion in ref. [23]). In the simple model we have presented, this would be achieved with an infrared behavior of the model gluon propagator of the form $\sim (\bar{k}^2)^\lambda$, with λ of order one, which corresponds to a typical order of magnitude appearing in those pure QCD scenarios where an infrared vanishing propagator for "confined" gluons is obtained.

As further studies suggested by the present work, we point out the consideration of similar nonperturbative effects in the fermion sector, as well as in other bag models where the hyperfine hadron structure is analyzed [24].

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